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Value creating stock manipulation: feedback effect of stock prices on firm value ☆

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Abstract

Higher stock prices relax a firm's budget constraint by increasing the value of its stock currency and enabling it to acquire larger targets through stock swaps. Higher prices also signal improved prospects to firm managers, potentially affecting their investment decisions and firm value. This feedback from prices to firm value complicates trading decisions of large traders, since their trades are likely to move prices. They need to condition not only on how their trades affect prices, but also on how these price changes induce further price changes through their impact on firm investments. This paper identifies equilibrium trading strategies of such traders and equilibrium price setting by a market maker. The question arises whether traders have an incentive to manipulate prices to get firms to undertake certain investments. The answer is yes, as long as traders maintain some inventory in the stock they trade. Moreover, the price manipulation turns out to be value-increasing for firms.

JEL classification: G10

Keywords: Manipulation; Feedback; Herding

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1. Introduction

A body of recent research has documented that, contrary to Modigliani-Miller's irrelevance principle, financial slack appears to impact firm investment decisions. This result has been established in surprisingly varied settings using different experiments and tests. Though not all issues related to endogeneity problems are resolved, the preponderance of evidence speaks clearly to the importance of financial slack.[1]

If financial slack is indeed important, any event, shock or strategy that affects financial slack will also affect firm value through changes in its investment set. This issue has been studied within the context of how financial constraints can amplify business cycles.[2] Starting with the assumption that budget constraints are binding, exogenous (usually macroeconomic) shocks which impact these constraints affect firm investment decisions, which in turn amplify these shocks further. Bernanke and Gertler (1989), for instance, show that an initial positive shock to the economy improves firms' profits and retained earnings. This allows firms to invest more, further increasing profits and retained earnings and amplifying the upturn.[3]

However, the shocks that affect financial constraints do not necessarily have to be macroeconomic or exogenous. Financial constraints can also be affected by what happens to a firm's stock price. Firms appear to take advantage of the higher prices of their stock by making more acquisitions. This phenomenon was particularly noticeable in the recent dot-com boom and viewed by the financial press as a firm leveraging its 'stock currency'.[4] There is also substantial evidence that firms raise more external funds when stock prices are high. Thus, higher stock prices appear to relax a firm's budget constraint, enlarging its investment set. Prices can also impact firm value if they signal information to firm managers and other stakeholders, allowing the latter to make more informed decisions. In Khanna et al. (1994) and Leland (1992), managers condition firm capacity choice on information inferred from stock price levels. In Subrahmanyam and Titman (2001), the 'feedback' from stock prices to firm cash flows occurs because stakeholders like employees, suppliers

[1] For an insightful review of this literature see Hubbard (1998).

[2] See Fisher (1933), Bernanke (1983), and Eckstein and Sinai (1986) among others for description of these 'accelerator-type' phenomena.

[3] Kiyotaki and Moore (1997) show a similar effect when asset values (as opposed to cash flows) are affected. A positive shock to land prices translates into increased borrowing capacity with land as collateral. The firm which owns land experiences a relaxation of its budget constraint and can make additional investments.

[4] Amazon.com made over 30 acquisitions between April 1998 and September 2000, when its stock price was between \$70 and \$207. No notable acquisition took place in the following 6 month period when its price plummeted. Nortel made about 20 acquisitions worth \$33.7 billion between 1997 and late 2000. 95 percent were made through stock swaps. Again, the acquisitions dried up when its stock price plummeted. The same is true of companies like Cisco, Intel, Dell and Microsoft. All made numerous acquisitions during this period and mainly through stock swaps. The acquisition activity slowed when their stock prices softened. Ironically, in 1999, Steven Case, CEO of AOL, referred to this as the era of 'free money' for internet firms and wondered what would happen to the internet industry when resources became tighter.

and customers condition on price levels when deciding whether to stay with the firm or leave.

We extend the analysis in the above literature to study the equilibrium trading behavior of informed traders in a setting with a feedback effect. If stock price movements alter a firm's investment set, decisions to make large trades that are likely to move stock prices become more complex. The price impact of such trades is driven not only by market microstructure effects, but also by their potential impact on firm decisions. Thus, large traders need to condition on not only how prices change in response to the information conveyed by their trades, but also on how these price changes induce further price changes through their impact on firms' investments. For similar reasons, the inference problem faced by the market maker also becomes more complex than in conventional microstructure models.

The ability of traders to affect a firm's investment decisions raises an interesting question: are there circumstances in which informed traders rationally manipulate trades in order to generate price patterns that increase a firm's value? For instance, if keeping prices high correctly signals to the firm to increase its capacity, would traders choose to buy shares (and keep prices high) even if their private signals suggest they should sell? With a rational market maker and small traders without inventories, the answer is no. The reason is that the market maker is aware of traders' incentives to create a particular price pattern to influence the firm's decisions in a particular way, and conditions on it when setting prices.[5] The market maker therefore sets prices where the informed trader makes a trading loss if the latter chooses to trade against her information.[6] So unless the trader can make profits elsewhere, she will not undertake this trade and the firm may lose a good investment opportunity.[7]

Conversely, if traders hold substantial inventories in the stock, the feedback effect enables them to recoup trading losses, provided they trade only a portion of their holding. Since the feedback effect changes firm value, an informed trader is willing to lose on the portion she trades as long as her remaining inventory increases in value by more than her trading loss. The larger her inventory and/or the larger the feedback effect, the more likely she is to manipulate her trade, and thus prices. It is worth noting that because of a positive inventory, an informed trader will manipulate her trade to induce the firm to increase investment only if it is expected to add value to the firm.

[5] This is in contrast to Subrahmanyam and Titman (2001) where the market maker is unaware of the manipulator and the strategies the latter would play in equilibrium. Here, established prices include the possibility of manipulation, ensuring that the manipulator makes negative trading profits when trading against her information. Thus, the feedback effect alone does not result in manipulation.

[6] Unlike in Grossman and Stiglitz (1980) the informed trader makes a profit if she trades in the direction of her signal. This happens because of noise traders in these types of models.

[7] Other papers have used similar arguments to support rational price manipulation or price bubbles. Allen and Gorton (1993) model agency problems between investors and fund managers. Even with an optimal contract, the bad fund manager rationally buys overvalued shares. He does not mind making losing trades as it does not adversely affect his compensation. Kumar and Seppi (1992) model a manipulator who intentionally loses in the spot market to improve his/her previously taken position in the futures market.

In our framework, we assume the trading pattern that triggers new firm investment consists of a sequence of three buy orders by informed traders. Suppose the first two traders buy. An interesting issue arises when the third informed trader gets information indicating she should sell. She realizes that the direction of her trade is pivotal to whether the firm will be able to accept a desirable new investment. If she buys and keeps prices high, the firm's investment set will increase. If she sells, the firm will forgo the incremental investment. She can deduce that the earlier two traders have information supporting buy orders. So in expectation, the project is a profitable one. Therefore it would appear that she should disregard her own information, and buy (i.e., 'herd' in the sense of an informational cascade).[8] However, the market maker anticipates the third trader's action, and prices accordingly, forcing the latter to take a loss if she trades against her information. In the absence of inventories, the third trader will not herd, and a potentially valuable investment is lost to the firm.

Some other results follow. Herding/manipulation is more likely when the quality of traders' information is higher. For higher quality information, the later informed are more likely to trust the information of those who have already traded and are more comfortable disregarding their own signal. For the same reason, the critical level of inventory needed to support herding as equilibrium behavior is inversely related to information quality. With higher information quality, the information loss from herding is less, and a smaller inventory is needed to recover the loss. Herding is more likely when the feedback effect is larger, and the critical level of inventory is negatively related to the magnitude of the feedback effect.

There is a region in our parameter space where we have multiple equilibria, i.e., both herding and non-herding equilibria can be supported. Though we cannot tell which will occur, we can derive some welfare implications. It turns out that the time $t = 0$ price is higher in the herding equilibrium than in the no-herding equilibrium. The reason is that herding gets the new project accepted with a higher probability, thus increasing expected fundamental value. However, since some information is lost in herding, the probability that the outcome will be good is reduced, which decreases fundamental value. In this region, the first effect appears to dominate. Interestingly, the unconditional expected price volatility is also higher under the herding equilibrium.

Herding also impacts the conditional volatility of price paths. On the extreme price-paths (ones with the highest volatility), herding dampens volatility, making price movements less extreme. Since these paths have 'bubble-like' characteristics, herding appears to moderate 'bubbles'.

We separate the notions of herding and momentum strategies. These have been interchangeably used in the literature, primarily because of the presumption that herding causes serial correlation in returns. However, in our model the market maker ensures that the price incorporates all his information at any point in time. Thus prices are martingales, and any period's price is an unbiased expectation of all

[8] Hence in our paper, 'manipulation' and 'herding' can be used interchangeably. We choose to use 'herding' because it appears to be more descriptive of how manipulation occurs in our setting.

possible future prices. Herding does not translate into momentum based trading profits.[9]

The remainder of this paper is organized as follows. Section 2 outlines the basic structure of the model. Section 3 identifies parameter spaces and conditions under which herding behavior might be supported in equilibrium. Section 4 considers another plausible equilibrium. In Section 5, we compare the two equilibria, and discuss some implications. Section 6 concludes the paper. The Appendix contains formal proofs of all results in the paper.

2. The model

2.1. Trading set-up

The model tries to capture the feedback effect as parsimoniously as possible. Assume a firm has existing assets with a state and trade dependent per share terminal payoff, v. Also assume there are only two states $\theta = H$ and $\theta = L$ (for 'High' and 'Low' respectively). Prior beliefs regarding the state are $\mathrm{P}(\theta = H) = \mathrm{P}(\theta = L) = \frac{1}{2}$. There are three risk-neutral informed traders: IT1, IT2 and IT3, each of whom can trade x_i shares, where $x_i \in X \equiv \{-1, +1\}$. Following Bikhchandani et al. (1992), each informed trader i is endowed with a (conditionally independent) signal s_i, which is an imperfect proxy for the state according to

	$\mathrm{P}(s_i = H \mid \theta)$	$\mathrm{P}(s_i = L \mid \theta)$
$\theta = H$	q	$1 - q$
$\theta = L$	$1 - q$	q

We need $q > \frac{1}{2}$, so that the signal is informative. Thus the probability of getting a H signal if the state is H, $\mathrm{P}(s_i = H \mid \theta = H) = q$, is higher than getting a L signal. Markets open for trading three times and, for tractability, only one informed trader trades each round and each can trade only once. The order in which they trade is randomly determined. At each round of trading, each informed trader is accompanied by one noise trader who randomly chooses between two equally likely trades $u_i \in U \equiv \{-1, +1\}$. There is a risk-neutral market maker who observes aggregate demand $y_i = x_i + u_i$ in each round of trading, and sets the share price such that his per-trade expected profits are zero, i.e., he sets a price equal to the conditional expectation of the value of a traded share, given the current history of

[9] However, because of the feedback effect, herding can be correlated with future asset prices through increase in inventory value. Thus, while our results are inconsistent with momentum in returns, they are consistent with findings in papers like Wermers (1999) that funds engaging in herd behavior outperform those that do not. They are also consistent with Falkenstein (1996), in that mutual funds might herd in stocks with specific characteristics, if one can assume that these characteristics proxy for the likelihood of a feedback effect.

information at that time.[10] This is a standard assumption in several models of market-making including Kyle (1985) and Glosten and Milgrom (1985).

Each informed trader can carry an inventory $I(>0)$ units of the firm's shares. While the market maker is aware that each informed trader carries some inventory, he does not know the exact value of I. As will be clear, whether the equilibrium consists of herding or no-herding is determined by common beliefs about the level of inventories.[11]

2.2. Security value and information structure

To capture the feedback effect simply, the state-trade-dependent terminal value of the share v, is modeled as follows. Assume that the only trading pattern that results in the firm changing its expected investment strategy consists of each informed trader unambiguously buying one share in each round of trading. This will happen when the trading pattern over three rounds is $\{+2, +2, +2\}$. For all other patterns of trade the firm's expected investments remain the same.[12] With unchanged expected investments, $v = 1$ in the H state and $v = 0$ in the L state. However, with the favorable trading pattern, the firm's expected investment set increases and the terminal payoffs are $v = 1 + d$ if the state is H and $v = -d$ if the state is L. Given uninformative signals (i.e., with $q = \frac{1}{2}$), the expected value of the firm remains at 0.5, with or without the new investment, but the variance of payoffs is higher with the new investment. However, when signals are informative (i.e., with $q > \frac{1}{2}$), indicating the High state as more likely, both expected value and payoff variance are higher.

The externality parameter d can be given a number of economic interpretations. As mentioned in the introduction, a high (and increasing) stock price increases the investment opportunity set of a firm. An implicit assumption here is that the firm's production technology has increasing returns to scale, which implies that all other things equal, a bigger firm has a greater (expected) value.[13] Another interpretation is that the firm gets a better deal in raising new funds, or in negotiating an acquisition, if prices remain high for the duration of these transactions. Thus, higher prices during this period contribute to increased firm value. Also, consistent with Subrahmanyam and Titman (2001), the feedback from stock price to firm cash flow could arise because a firm's non-financial stakeholders condition their decisions on information revealed by the price.

[10] It may be observed that similar results to ours would obtain if the market maker could observe the two components of the order flow individually, but could not identify the trader submitting the orders as informed or not. We thank an anonymous referee for pointing this out to us.

[11] In modeling informed traders with inventories, we have in mind large institutions who have the ability to influence prices with the size of their trades. Note that the empirical literature in finance has identified precisely such institutions engaging in herd behavior.

[12] The main conclusions are not affected by which trading pattern signals better opportunities. The effect would be captured by any trading pattern that suggests that the probability of the High state is greater.

[13] We are thankful to an anonymous referee for bringing this point to our attention.

The set of final payoffs and beliefs is summarized as follows.

a. $v = 1 + d$, if $\theta = H$ and the trading pattern is $\{+2, +2, +2\}$ over the three rounds. This occurs when each informed trader and each noise trader buys one share in all three rounds of trading.
   $v = -d$, if $\theta = L$ and the trading pattern is $\{+2, +2, +2\}$ over the three rounds.
   $v = 1$ if $\theta = H$, and $v = 0$, if $\theta = L$, with any other combination of trades over the three rounds.[14]
b. All players know the information regarding the terminal price of the share, and the precise value of d (> 0).

The following timeline summarizes the sequence of actions that take place in our model.

0	1	2	3	end
All know the values of q and d and the distribution of terminal payoff v across states and paths	*Round* 1 *of trading* Market maker observes $y_1 = x_1 + u_1$ and sets p^1	*Round* 2 *of trading* Market maker observes $y_2 = x_2 + u_2$ and sets p^2	*Round* 3 *of trading* Market maker observes $y_3 = x_3 + u_3$ and sets p^3	State θ and terminal value v of share realized

2.3. *Equilibrium*

We now prepare to search for a herding equilibrium under the current set-up. This serves two purposes. First, it illustrates the model in detail. Second, it also answers the paper's main question: Are there circumstances in which informed traders rationally manipulate trades in order to generate price patterns that increase a firm's value?

We restrict the focus to a symmetric Nash equilibrium.[15] A sequence of trading strategies and a belief system constitute an equilibrium, if, given the belief system and the strategies of other players, no player has an incentive to deviate from her

[14] Note that the externality '*d*' occurs when there are several *trades* in a similar direction. Intuitively, we want to capture the feedback effect of *prices*, where higher prices lead to a broader opportunity set for the firm. However, in a model such as ours, and in models of market microstructure such as Kyle (1985) and Subrahmanyam (1991), conditioning on price and net order flow is the same thing. The conclusions from this simple set-up are robust to alternative ways of modeling the resource allocation role of prices.

[15] Technically speaking, we employ the concept of a Bayesian Nash equilibrium with the intuitive criterion.

prescribed strategy. For tractability, we confine ourselves to the following definition of a herding equilibrium.[16]

Definition 1. A herding equilibrium is defined as one where at least one informed trader chooses to trade in a direction similar to the earlier trades, but against her own private signal.

Our definition of herding is consistent with naive price-momentum or trend-chasing strategies as documented in the recent empirical literature. From the above definition, it is obvious that the first informed trader always follows her signal, as she has no history of trades to consider. The best she can do is to trade $x_1 = +1$ if $s_1 = H$, and $x_1 = -1$ if $s_1 = L$. As the following proposition shows, herding never benefits the second informed trader either. However, the third informed trader might find it sometimes beneficial to herd.

Proposition 1. *It does not ever benefit the first two informed traders to herd but it does benefit the third informed trader to herd when she can conclusively observe that the first two traders have traded in the same direction.*

Intuitively, informed trader i is 'correct' (ex-post) if she traded $x_i = +1$ and the state is revealed to be $\theta = H$, or if she traded $x_i = -1$ and the state is revealed to be $\theta = L$. Otherwise she is 'wrong' ex-post. The proof of the above proposition follows from the fact that herding improves the probability that the third trader is correct regarding the terminal state, when both previous signals are identical. This is because the two earlier signals (if unambiguous) dominate her own signal, even if it is opposite in sign. Hence, only the third player will ever herd.

3. Herding as equilibrium behavior

3.1. Herding equilibrium

Proposition 1 suggests a candidate equilibrium with herding. However, this equilibrium needs to survive price-setting by a rational market maker. Thus, we first derive prices under the belief that herding will occur. Next, we verify whether the informed traders find it profitable to herd, given these prices.

Table 1 presents the prices established under the belief of herding.[17] (Also, see Table 2 for combinations of signals and trades in a herding equilibrium.) Given the

[16] This is similar to the approach in Khanna (1998). We later identify conditions that support this restriction on choice of candidate. It turns out that only the conditions on the third informed trader are binding.

[17] In Table 1, p^i_j represents a price in round i of trading, following history j in previous rounds. i can take values 1, 2 or 3, while j can take -2, 0 or $+2$ (aggregate demand level) for each round of trading. For example, $p^2_{0,-2}$ means that this is a price established in the second round of trading following an aggregate demand of 0 in the first round of trading, and -2 in the second round of trading.

Table 1
Prices under herding

Price at time $t = 0$

$p^0 = \frac{1}{2} + \frac{d(2q-1)}{16}$

Round 1 *of trading*

$p^1_{-2} = (1-q)$

$p^1_0 = \frac{1}{2}$

$p^1_2 = q + \frac{d(2q-1)}{4}$

Round 2 *of trading*

$p^2_a \equiv p^2_{-2,-2} = \frac{(1-q)^2}{q^2+(1-q)^2}$

$p^2_b \equiv p^2_{-2,0} = p^2_{0,-2} = (1-q)$

$p^2_c \equiv p^2_{-2,2} = p^2_{2,-2} = p^2_{0,0} = \frac{1}{2}$

$p^2_d \equiv p^2_{0,2} = p^2_{2,0} = q$

$p^2_e \equiv p^2_{2,2} = \frac{q^2}{q^2+(1-q)^2} + \frac{d}{2}\frac{2q-1}{q^2+(1-q)^2}$

Round 3 *of trading*

$p^3_{a,-2} = \frac{(1-q)^3}{q^3+(1-q)^3}$	$p^3_{a,0} = \frac{(1-q)^2}{q^2+(1-q)^2}$	$p^3_{a,2} = (1-q)$
$p^3_{b,-2} = \frac{(1-q)^2}{q^2+(1-q)^2}$	$p^3_{b,0} = (1-q)$	$p^3_{b,2} = \frac{1}{2}$
$p^3_{c,-2} = (1-q)$	$p^3_{c,0} = \frac{1}{2}$	$p^3_{c,2} = q$
$p^3_{d,-2} = \frac{1}{2}$	$p^3_{d,0} = q$	$p^3_{d,2} = \frac{q^2}{q^2+(1-q)^2}$
$p^3_{e,-2} = q$	$p^3_{e,0} = \frac{q^2}{q^2+(1-q)^2}$	$p^3_{e,2} = \frac{q^2}{q^2+(1-q)^2} + \frac{d(2q-1)}{q^2+(1-q)^2}$

Table 2
Beliefs and outcomes in herding equilibrium. Since the only sequence of trades that results in the firm changing its expected investment strategy is $+2, +2, +2$, we show in this table only those combinations of signals and trades that could potentially result in this sequence. All other combinations of signals and trades are the same for both the herding and no-herding equilibria

s_1	x_1	y_1	s_2	x_2	y_2	s_3	x_3	y_3	Value v	in State
									H	L
H	$+1$	$+2$	H	$+1$	$+2$	H	$+1$	$+2$	$1+d$	$-d$
						H	$+1$	0	1	0
						L	$+1$	$+2$	$1+d$	$-d$
						L	$+1$	0	1	0

derived prices, it remains to check whether the informed traders find it profitable to adopt the conjectured equilibrium strategies rather than deviate. The following result can then be established.

Proposition 2. *There exists a herding equilibrium if, and only if, informed traders have inventories* $I > I^*$, *where*

$$I^* = \frac{3}{d(2q-1)}\left[\frac{q^2}{q^2+(1-q)^2} - q\right] + \left[\frac{1}{q^2+(1-q)^2} - 1\right], \quad \forall q \in (0.5, 1],$$

and the market maker believes that each informed trader has an inventory $I > I^*$.

The criticality of both the feedback effect of prices and the inventory assumption in our model is evident from Proposition 2. In the absence of feedback (i.e., if the quantity d is zero), the third informed trader cannot offset trading losses with gains on her inventory. Alternatively, as can be seen from the expression for I^*, the minimum required inventory becomes very large as d goes to zero.

The other important implication of Proposition 2 is that in the absence of inventories, herd behavior cannot be supported in equilibrium. This implication is similar to Proposition 2 of Avery and Zemsky (1998). In the presence of a rational market maker who sets prices on the basis of net order flow and the belief system, uncertainty regarding the state is not enough to induce herding behavior. In our model, as in theirs, the market maker, anticipating herding by the third informed trader, sets prices at a level at which it is unprofitable (in expectation) for the latter to go against an L signal and buy. The reason is that the market maker sets the third period price knowing that the third trader will buy independently of his signal. Thus he pools over the trader's next period's signal being High or Low with equal probability. If the third informed trader gets a Low signal, the price she faces when she buys exceeds her expectation of the terminal value. If she buys, she makes an expected loss. Consequently, herding cannot be supported in equilibrium for any set of parameters.

In the presence of both feedback and inventories, even though the third informed trader makes a trading loss if she herds, she stands to gain on her inventory of securities since the expected terminal value of her shares is higher under herding. In other words, she knows that by herding she will create an externality and as long as she has a sufficient stake in the externality, she will choose to do so.

3.2. Robustness of the herding equilibrium

A natural question to ask at this juncture is whether herding is possible when the signals are totally uninformative. After all, if there are gains to herding (in the form of the externality that is created), totally uninformed traders might decide to start herding by trading $x_i = +1$. Corollary 1 provides the answer to this question, along with simple comparative statics on the minimum inventory bound I^*.

Corollary 1. a. *With totally uninformative signals, i.e., with* $q = 0.5$, *a herding equilibrium is not possible.*

b. *For given* d, I^* *decreases in* q, $\forall q > 0.5$.

c. $\forall q \in (0.5, 1]$, I^* *decreases in* d, $\forall d > 0$.

4. A no-herding equilibrium

To facilitate comparison with the herding case, it is useful to establish conditions under which a no-herding equilibrium exists. Note that the additional investment still occurs after a $\{+2, +2, +2\}$ pattern of trades. Without herding, the (ex-ante) probability of the additional investment occurring is lower as the third informed trader trades $x_3 = +1$ only if her signal is H. Recall that with herding, she trades $x_3 = +1$ independently of her signal. However, there is also a counter effect. In the no-herding case the additional investment occurs conditional on three H signals. In contrast, in the herding equilibrium the externality occurs conditional on only two H signals, as the third trader's signal is irrelevant. This means that, in the no-herding case, it is more likely that the terminal state will be 'High'. Thus, from a social welfare angle, it is not clear in which equilibrium society is better off. We next attempt to answer this question.

Definition 2. A no-herding equilibrium is defined as one where every informed trader always trades following her own signal.

Table 3 presents the prices established under the beliefs of no-herding.[18] (See Table 4 for combinations of signals and trades in a no-herding equilibrium.) Once again, as for the herding case, given the prices, we need to check whether the informed traders find it profitable to adopt the conjectured equilibrium strategies rather than to deviate (and herd.) The following result can be established.

Proposition 3. *A no-herding equilibrium exists if, and only if, each informed trader has inventory $I < \tilde{I}$, where*

$$\tilde{I} = \left[\frac{2q^2}{d(2q-1)[q^2 + (1-q)^2]} + \frac{q^3}{d(2q-1)[q^3 + (1-q)^3]} - \frac{3q}{d(2q-1)} + \frac{q^3 - (1-q)^3}{(2q-1)[q^3 + (1-q)^3]} - 1 \right]$$

and the market maker believes that each informed trader has an inventory $I < \tilde{I}$.

Intuitively, Proposition 3 says that for each informed trader to follow her own signal, her individual inventory should be sufficiently small. If inventories are large, then informed traders will be tempted to go against their L signals and buy in the hope of creating the feedback effect, and making profits on their inventories. But this deviation on their part causes the equilibrium to break down.

[18] In Table 3, p_j^{ni} represents a price in round i of trading, following history j in previous rounds. i can take values 1, 2 or 3, while j can take -2, 0, $+2$ (aggregate demand level) for each round of trading. The n in the superscript refers to the fact that these prices are derived under the belief system of no-herding.

Table 3
Prices under no-herding

Price at time $t = 0$

$p^{n0} = \frac{1}{2} + \frac{d[q^3-(1-q)^3]}{16}$

Round 1 *of trading*

$p^{n1}_{-2} = (1-q)$

$p^{n1}_{0} = \frac{1}{2}$

$p^{n1}_{2} = q + \frac{d[q^3-(1-q)^3]}{4}$

Round 2 *of trading*

$p^{n2}_{a} \equiv p^{n2}_{-2,-2} = \frac{(1-q)^2}{q^2+(1-q)^2}$

$p^{n2}_{b} \equiv p^{n2}_{-2,0} = p^{n2}_{0,-2} = (1-q)$

$p^{n2}_{c} \equiv p^{n2}_{-2,2} = p^{n2}_{2,-2} = p^{n2}_{0,0} = \frac{1}{2}$

$p^{n2}_{d} \equiv p^{n2}_{0,2} = p^{n2}_{2,0} = q$

$p^{n2}_{e} \equiv p^{n2}_{2,2} = \frac{q^2}{q^2+(1-q)^2} + \frac{d}{2}\frac{q^3-(1-q)^3}{q^2+(1-q)^2}$

Round 3 *of trading*

$p^{n3}_{a,-2} = \frac{(1-q)^3}{q^3+(1-q)^3}$	$p^{n3}_{a,0} = \frac{(1-q)^2}{q^2+(1-q)^2}$	$p^{n3}_{a,2} = (1-q)$
$p^{n3}_{b,-2} = \frac{(1-q)^2}{q^2+(1-q)^2}$	$p^{n3}_{b,0} = (1-q)$	$p^{n3}_{b,2} = \frac{1}{2}$
$p^{n3}_{c,-2} = (1-q)$	$p^{n3}_{c,0} = \frac{1}{2}$	$p^{n3}_{c,2} = q$
$p^{n3}_{d,-2} = \frac{1}{2}$	$p^{n3}_{d,0} = q$	$p^{n3}_{d,2} = \frac{q^2}{q^2+(1-q)^2}$
$p^{n3}_{e,-2} = q$	$p^{n3}_{e,0} = \frac{q^2}{q^2+(1-q)^2}$	$p^{n3}_{e,2} = \frac{q^3}{q^3+(1-q)^3} + \frac{d[q^3-(1-q)^3]}{q^3+(1-q)^3}$

Table 4
Beliefs and outcomes in no-herding equilibrium. Since the only sequence of trades that results in the firm changing its expected investment strategy is +2, +2, +2, we show in this table only those combinations of signals and trades that could potentially result in this sequence. All other combinations of signals and trades are the same for both the herding and no-herding equilibria

s_1	x_1	y_1	s_2	x_2	y_2	s_3	x_3	y_3	Value v	in State
									H	L
H	+1	+2	H	+1	+2	H	+1	+2	$1+d$	$-d$
						H	+1	0	1	0
						L	−1	−2	1	0
						L	−1	0	1	0

5. Multiple equilibria

Comparing I^* (which is the lower bound on inventory in the herding equilibrium) and $\tilde{I}$ (the upper bound on inventory in the no-herding case), we can see that for the

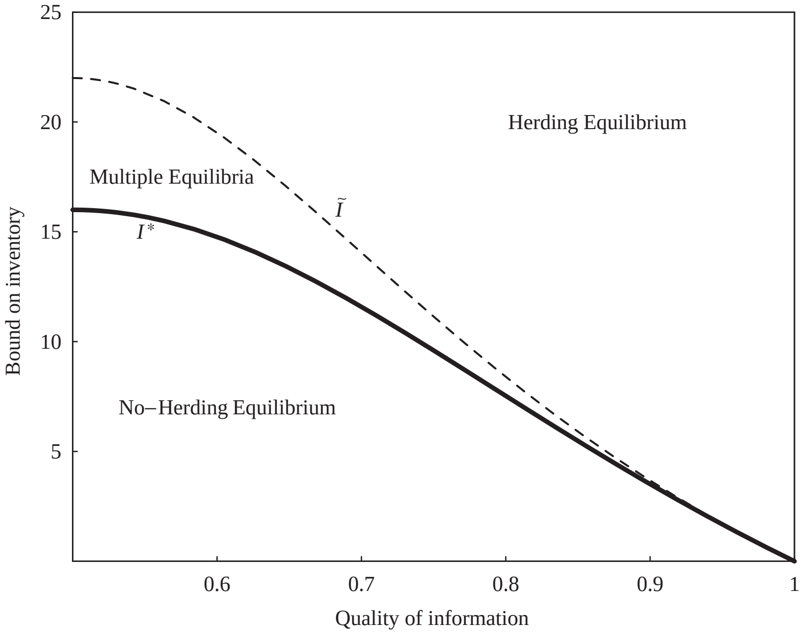


Fig. 1. Range of inventory for multiple equilibria ($d = 0.1$). $\tilde{I}$ is the maximum bound on inventory for a no-herding equilibrium to exist. I^* is the minimum bound on inventory for a herding equilibrium to exist. It can be seen from the above figure that over the support of q that is relevant, i.e., for $q \in (0.5, 1]$, there is always an intermediate range of inventory for which multiple equilibria exist.

relevant support of q that we consider, i.e., for $q \in (0.5, 1]$, $\tilde{I}$ is always greater than I^*. While this is not immediately obvious from the expressions for the inventory bounds, it can be seen from Fig. 1 that there is always an intermediate range of inventory for which both equilibria are possible. Though we cannot tell which equilibrium will occur, we are afforded the opportunity of making some interesting comparisons.

The region for which multiple equilibria exist, while widest for low q (see Fig. 1), increases fastest for the intermediate q's. The reason is that the information loss from herding, $\mathrm{P}(\theta = H \mid s_1 = H, s_2 = H, s_3 = H) - \mathrm{P}(\theta = H \mid s_1 = H, s_2 = H, s_3 = H \text{ or } L)$, is greatest at the intermediate quality of information. For low values of q the information loss from herding is small because the signal is not particularly informative. For high values of q the marginal contribution of the third signal is low since there is little noise in the first two signals. Thus, the slopes of the lower and upper inventory bounds are steepest at these values.

The conditional volatility of the price paths also exhibits a number of regularities. It turns out that on the extreme price paths, the ones with the highest volatility, herding serves to dampen volatility, thus making price movements less extreme. (See Figs. 2(a) and (b)). The reason lies in the way the market maker sets prices under the two equilibria. The figures show that, not only is p^0 higher with herding, but so are p^1 and p^2. The initial price p^0 is higher because the probability that the additional

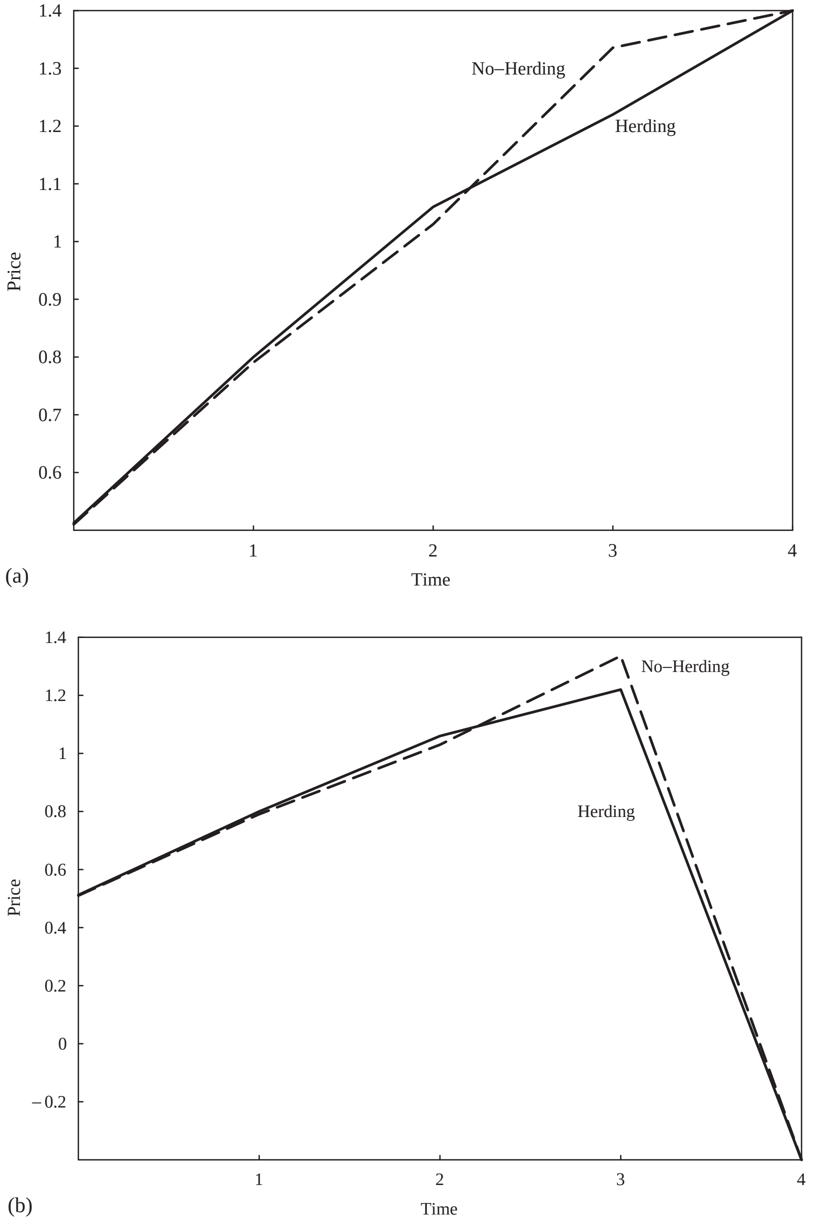


Fig. 2. (a) Price path along the trade sequence $\{+2, +2, +2\}$ with H realization ($q = 0.75,\ d = 0.4$), (b) Price path along the trade sequence $\{+2, +2, +2\}$ with L realization ($q = 0.75,\ d = 0.4$).

investment will occur is higher with herding. First and second round prices p^1 and p^2 are higher both because of the higher probability of investment and because the probability of the H state is higher (after two rounds of trading.) However, the third

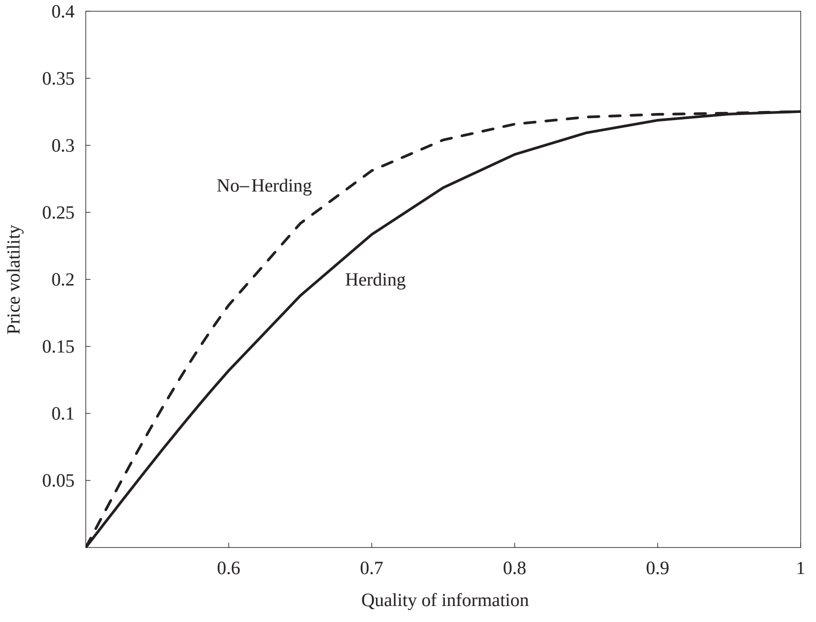
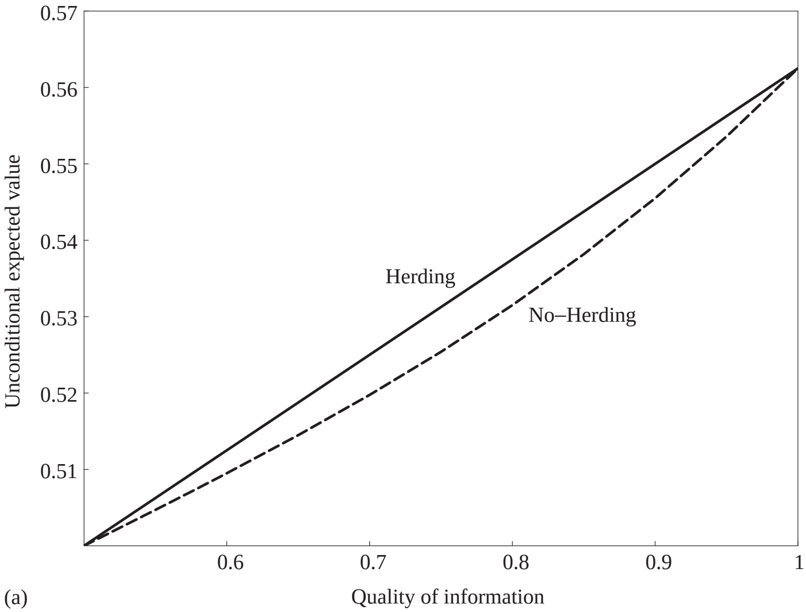


Fig. 3. Volatility (time-series standard deviation) of prices along the price path $\{p^0, p^1_2, p^2_{2,2}, p^3_{2,2,2}\}$ $(d = 0.4)$.

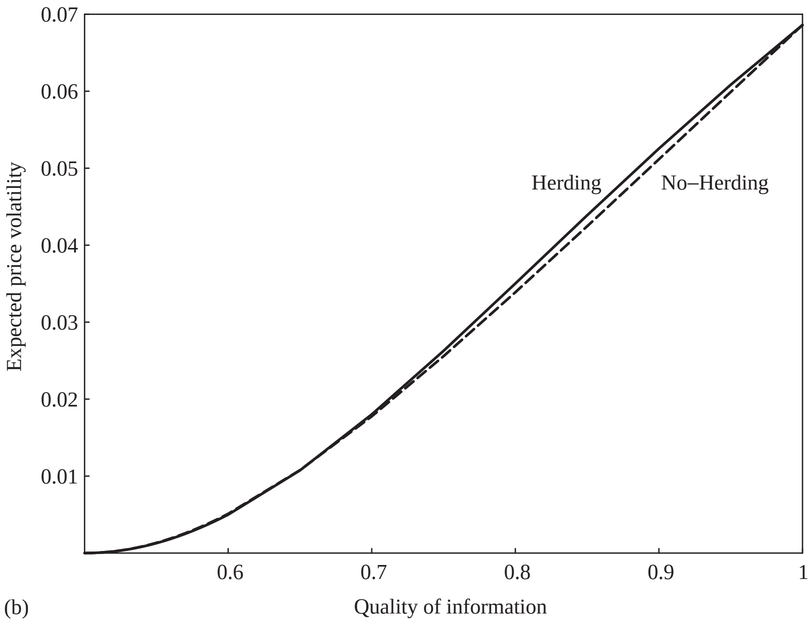
round price, p^3 is lower in the herding equilibrium because the market maker cannot infer the third trader's signal under herding since the latter buys independently of her signal. Consequently, the market maker averages over the expected value of the firm with $s_3 = L$ and $s_3 = H$. In the no-herding equilibrium, however, the price is higher as it is unambiguously conditioned on three H signals.

Since these extreme paths have 'bubble-like' characteristics, herding can be argued to be moderating 'bubble-like' phenomena. This pattern is more obvious in Fig. 2(b). One can see that when the 'bubble' bursts, the drop in price is less under herding. From Fig. 3 one can observe that the conditional volatility of the extreme price path is uniformly less for the herding equilibrium, irrespective of the quality of information.

As mentioned above, the time $t = 0$ price is higher for the herding equilibrium than for the no-herding equilibrium. Compare the values of time 0 prices in Tables 1 and 3 of the text. (Also see Fig. 4(a).) The reason is that herding gets the new project accepted with a higher probability, thus increasing expected fundamental value. However, since some information is lost in herding, the probability that the outcome will be good given a pattern of trades is reduced, which decreases fundamental value. In this region, though, the first effect appears to dominate, resulting in a higher value with herding. However, the expected volatility of prices (with the expectation taken over all 27 possible paths) is also higher under herding, as can be seen from Fig. 4(b).



(a)



(b)

Fig. 4. (a) Unconditional expected security value: herding vs. no-herding ($d = 1.0$). The unconditional expected security value is the price that would prevail at time $t = 0$ in each equilibrium. (b) Unconditional expected volatility: herding vs. no-herding ($d = 1.0$). We first calculate the variance of the four prices (p^0, p^1, p^2 and p^3) for each possible path, and then calculate a (probability) weighted average of such variances over all the 27 paths. Underlying this calculation is a frequentist interpretation of the probabilities of occurrence of the 27 paths.

6. Conclusion

In this paper we establish price manipulation and herding in a rational expectations framework. When price paths influence a firm's investment decisions, endowing informed investors with positive inventories results in price manipulation. The implication for existing noise trading models (e.g., DeLong et al., 1990) or behavioral models such as Barberis et al. (1998), Daniel et al. (1998), and Hong and Stein (1999) is that an additional source of uncertainty probably needs to be priced: price movements result in changes to the real size and underlying riskiness of the firm's assets.

In our framework, herding by institutions or funds increases fundamental value and is unlikely to be destabilizing.[19] Also, volatilities along extreme price paths are dampened because rational price makers cannot perfectly infer investors' signals from their trades and thus pool over them. Investors are hurt less when bad outcomes occur.

The model outlined here can be extended in several directions in order to analyze a rich set of open questions. Although we have not formulated a general equilibrium model here, it is conceivable that our theory could be extended to provide an alternative explanation for the 'excessive' volatility documented by Shiller (1981) and LeRoy and Porter (1981). If prices are used by markets, among other things, to effect changes in real firm investments, stock volatility will in part be a function of how frequently markets try to impact a firm's investment decisions. This also explains why trading may generate volatility. If price movements facilitate changes in firm values, one would expect higher price volatility when markets are open and trades are affecting prices. Also, it helps explain why firms appear to be 'obsessively' concerned about their stock price, and can justify stock price charting as a seemingly relevant profession.

Finally, in our model, rational herding does not translate into momentum profits. As in other rational expectation models, our prices have no predictive power over future prices. The market maker conditions both on the feedback effect and the possibility of herding when establishing prices. Whether this model can be extended to result in momentum profits is left to future research.

Appendix A

A.1. Prices under herding: Expressions in Table 1

Price setting: *round* 1 *of trading*:

$$p^1_{-2} = \mathrm{P}(\theta = H \mid s_1 = L) = (1 - q),$$

[19] Studies like Friend et al. (1970) and Lakonishok et al. (1992) have reported that in an apparent attempt to take advantage of momentum profits, large traders follow similar or 'herd-like' trading strategies: buying when other participants are buying, and selling when others are selling. Also see Grinblatt et al. (1995) and Nofsinger and Sias (1999) among others.

$$p_0^1 = \tfrac{1}{2}[\mathrm{P}(\theta = H \mid s_1 = H) + \mathrm{P}(\theta = H \mid s_1 = L)] = \tfrac{1}{2},$$

$$\begin{aligned} p_2^1 = {} & \mathrm{P}(s_2 = L \;\&\; \theta = H \mid s_1 = H) + \mathrm{P}(s_2 = H \;\&\; y_2 = 0 \;\&\; \theta = H \mid s_1 = H) \\ & + (1+d)\mathrm{P}(s_2 = H \;\&\; y_2 = 2 \;\&\; s_3 = H \;\&\; y_3 = 2 \;\&\; \theta = H \mid s_1 = H) \\ & - d\mathrm{P}(s_2 = H \;\&\; y_2 = 2 \;\&\; s_3 = H \;\&\; y_3 = 2 \;\&\; \theta = L \mid s_1 = H) \\ & + (1+d)\mathrm{P}(s_2 = H \;\&\; y_2 = 2 \;\&\; s_3 = L \;\&\; y_3 = 2 \;\&\; \theta = H \mid s_1 = H) \\ & - d\mathrm{P}(s_2 = H \;\&\; y_2 = 2 \;\&\; s_3 = L \;\&\; y_3 = 2 \;\&\; \theta = L \mid s_1 = H) \\ & + \mathrm{P}(s_2 = H \;\&\; y_2 = 2 \;\&\; s_3 = H \;\&\; y_3 = 0 \;\&\; \theta = H \mid s_1 = H) \\ & + \mathrm{P}(s_2 = H \;\&\; y_2 = 2 \;\&\; s_3 = L \;\&\; y_3 = 0 \;\&\; \theta = H \mid s_1 = H) \\ = {} & q + \frac{d}{4}(2q-1). \end{aligned}$$

From the above three prices, it is easy to derive the price at time 0 that would prevail in this equilibrium.

$$\begin{aligned} p^0 = {} & p_{-2}^1 \mathrm{P}(y_1 = -2) + p_0^1 \mathrm{P}(y_1 = 0) + p_2^1 \mathrm{P}(y_1 = 2) \\ = {} & \frac{1}{2} + \frac{d}{16}(2q-1). \end{aligned}$$

Price setting: *round* 2 *of trading*:

$$\begin{aligned} p_{-2,-2}^2 &= \mathrm{P}(\theta = H \mid s_1 = L \;\&\; s_2 = L) = \frac{(1-q)^2}{q^2 + (1-q)^2}, \\ p_{-2,0}^2 &= \mathrm{P}(\theta = H \mid s_1 = L) = (1-q), \\ p_{-2,2}^2 &= \mathrm{P}(\theta = H \mid s_1 = L \;\&\; s_2 = H) = \tfrac{1}{2}, \\ p_{0,-2}^2 &= \mathrm{P}(\theta = H \mid s_2 = L) = (1-q), \\ p_{0,0}^2 &= \mathrm{P}(\theta = H) = \tfrac{1}{2}, \\ p_{0,2}^2 &= \mathrm{P}(\theta = H \mid s_2 = H) = q, \\ p_{2,-2}^2 &= \mathrm{P}(\theta = H \mid s_1 = H \;\&\; s_2 = L) = \tfrac{1}{2}, \\ p_{2,0}^2 &= \mathrm{P}(\theta = H \mid s_1 = H) = q, \end{aligned}$$

$$\begin{aligned} p_{2,2}^2 = {} & (1+d)\mathrm{P}(s_3 = H \;\&\; y_3 = 2 \;\&\; \theta = H \mid s_1 = H \;\&\; s_2 = H) \\ & - d\mathrm{P}(s_3 = H \;\&\; y_3 = 2 \;\&\; \theta = L \mid s_1 = H \;\&\; s_2 = H) \\ & + \mathrm{P}(s_3 = H \;\&\; y_3 = 0 \;\&\; \theta = H \mid s_1 = H \;\&\; s_2 = H) \\ & + (1+d)\mathrm{P}(s_3 = L \;\&\; y_3 = 2 \;\&\; \theta = H \mid s_1 = H \;\&\; s_2 = H) \\ & - d\mathrm{P}(s_3 = L \;\&\; y_3 = 2 \;\&\; \theta = L \mid s_1 = H \;\&\; s_2 = H) \\ & + \mathrm{P}(s_3 = L \;\&\; y_3 = 0 \;\&\; \theta = H \mid s_1 = H \;\&\; s_2 = H) \\ = {} & \frac{q^2}{q^2 + (1-q)^2} + \frac{d}{2}\left[\frac{2q-1}{q^2 + (1-q)^2}\right]. \end{aligned}$$

Price setting: round 3 of trading:

$$p^3_{a,-2} = p^3_{-2,-2,-2} = \mathrm{P}(\theta = H \mid s_1 = L \ \& \ s_2 = L \ \& \ s_3 = L) = \frac{(1-q)^3}{q^3 + (1-q)^3},$$

$$p^3_{a,0} = p^3_{-2,-2,0} = \mathrm{P}(\theta = H \mid s_1 = L \ \& \ s_2 = L) = \frac{(1-q)^2}{q^2 + (1-q)^2},$$

$$p^3_{a,2} = p^3_{-2,-2,2} = \mathrm{P}(\theta = H \mid s_1 = L \ \& \ s_2 = L \ \& \ s_3 = H) = (1-q),$$

$$p^3_{b,-2} = p^3_{-2,0,-2} = \mathrm{P}(\theta = H \mid s_1 = L \ \& \ s_3 = L) = \frac{(1-q)^2}{q^2 + (1-q)^2},$$

$$p^3_{b,0} = p^3_{-2,0,0} = \mathrm{P}(\theta = H \mid s_1 = L) = (1-q),$$

$$p^3_{b,2} = p^3_{-2,0,2} = \mathrm{P}(\theta = H \mid s_1 = L \ \& \ s_3 = H) = \tfrac{1}{2},$$

$$p^3_{c,-2} = p^3_{-2,2,-2} = p^3_{0,0,-2} = \mathrm{P}(\theta = H \mid s_3 = L) = (1-q),$$

$$p^3_{c,0} = p^3_{-2,2,0} = p^3_{0,0,0} = \mathrm{P}(\theta = H) = \tfrac{1}{2},$$

$$p^3_{c,2} = p^3_{-2,2,2} = p^3_{0,0,2} = \mathrm{P}(\theta = H \mid s_3 = H) = q,$$

$$p^3_{d,-2} = p^3_{0,2,-2} = p^3_{2,0,-2} = \mathrm{P}(\theta = H \mid s_1 = H \ \& \ s_3 = L) = \tfrac{1}{2},$$

$$p^3_{d,0} = p^3_{0,2,0} = p^3_{2,0,0} = \mathrm{P}(\theta = H \mid s_1 = H) = q,$$

$$p^3_{d,2} = p^3_{0,2,2} = p^3_{2,0,2} = \mathrm{P}(\theta = H \mid s_1 = H \ \& \ s_3 = H) = \frac{q^2}{q^2 + (1-q)^2},$$

$$p^3_{e,0} = p^3_{2,2,0} = \mathrm{P}(\theta = H \mid s_1 = H \ \& \ s_2 = H) = \frac{q^2}{q^2 + (1-q)^2},$$

$$\begin{aligned} p^3_{e,2} = p^3_{2,2,2} &= (1+d)\mathrm{P}(\theta = H \mid s_1 = H \ \& \ s_2 = H \ \& \ s_3 = H \text{ or } L) \\ &\quad - d\mathrm{P}(\theta = L \mid s_1 = H \ \& \ s_2 = H \ \& \ s_3 = H \text{ or } L) \\ &= \frac{q^2}{q^2 + (1-q)^2} + d\left[\frac{2q-1}{q^2 + (1-q)^2}\right]. \end{aligned}$$

Under the proposed herding equilibrium, $p^3_{e,-2} = p^3_{2,2,-2}$ is off the equilibrium path. Here we use the intuitive criterion (Cho and Kreps, 1987) to interpret this sequence of events, i.e., the market maker assumes that the third informed trader made a mistake in moving off-equilibrium by not herding, but that such a mistake is more likely if she had an L signal rather an H signal.

Therefore, $p^3_{e,-2} = \mathrm{P}(\theta = H \mid s_1 = H \ \& \ s_2 = H \ \& \ s_3 = L) = q$.

This completes the derivation of the prices in Table 1.

Proposition 1. *It does not ever benefit the first two informed traders to herd but it does benefit the third informed trader to herd when she can conclusively observe that the first two traders have traded in the same direction.*

Proof. This proof is almost identical to the proof of Proposition 1 in Khanna (1998). We first show that the first and second informed traders strictly decrease the probability of being 'correct' ex-post, by ignoring their own private signal, and then proceed to show that it could benefit the third informed trader to herd in some cases. Note that as stated in the body of the paper, informed trader i is 'correct' ex-post if she traded $x_i = +1$ and the state is revealed to be $\theta = H$, or if she traded $x_i = -1$ and the state is revealed to be $\theta = L$. Table A1 sets out the (posterior) probabilities of being correct and wrong for each informed trader, with the available choice of trades.

Informed trader 1: It can be easily verified that, for both $s_1 = H$ and $s_1 = L$, IT1 strictly decreases her probability of being correct by ignoring her own signal. For instance, when $s_1 = L$, following her signal and choosing $x_1 = -1$ implies a probability of being correct of $P(\theta = L \mid s_1 = L) = q$, while ignoring her signal and choosing $x_1 = +1$ implies a probability of being correct of $P(\theta = H \mid s_1 = L) = 1 - q$. The case $s_1 = L$ can be dealt with in similar fashion.

Informed trader 2: Table A2 facilitates the calculation of the posterior probabilities in this case. Check the first row. At this point, we know that the first informed had a signal H, and had chosen $x_1 = +1$. (IT2, of course, knows this for sure because she observes $y_1 = +2$.) IT2 has a H signal. Thus, if she goes with her signal and chooses $x_2 = +1$, then her probability of being correct is $P(\theta = H \mid s_1 = H, s_2 = H)$, which, by an application of Bayes' rule, can be calculated as

$$\frac{P(s_1 = H, s_2 = H \mid \theta = H)P(\theta = H)}{P(s_1 = H, s_2 = H)} = 0.5q^2/P(s_1 = H, s_2 = H).$$

If she ignores her signal (and chooses $x_2 = -1$), then her probability of being correct is $P(\theta = L \mid s_1 = H, s_2 = H)$, which can be calculated as $0.5(1-q)^2/P(s_1 = H,$

Table A1
Generic table of posterior probabilities

	IT i's trade	
	$x_i = +1$	$x_i = -1$
Prob. (correct)	$P(\theta = H \mid s_j = H, j \neq i, s_i)$	$P(\theta = L \mid s_j = H, j \neq i, s_i)$
Prob. (wrong)	$P(\theta = L \mid s_j = H, j \neq i, s_i)$	$P(\theta = H \mid s_j = H, j \neq i, s_i)$

Table A2
Decisions and probabilities: first two informed traders

Action/signal of IT1	IT2's signal	Prob. that the signals come from	
		State $\theta = H$	State $\theta = L$
$x_1 = +1/s_1 = H$	H	q^2	$(1-q)^2$
	L	$q(1-q)$	$q(1-q)$
$x_1 = -1/s_1 = L$	H	$q(1-q)$	$q(1-q)$
	L	$(1-q)^2$	q^2

Table A3
Decisions and probabilities: all three informed traders

Action and signal		Signal IT3	Prob. signals come from	
IT1	IT2		State $\theta = H$	State $\theta = L$
$x_1 = +1/s_1 = H$	$x_2 = +1/s_2 = H$	H	q^3	$(1-q)^3$
		L	$q^2(1-q)$	$q(1-q)^2$

$s_2 = H$). Since $q > 0.5$, she is better off following her signal. It can be seen that for the second and third rows, she is indifferent, while for the fourth row, her probability of being correct is again higher, if she chooses $x_2 = -1$, following her L signal.

Informed trader 3: Table A3 will facilitate the calculation of the posterior probabilities in the case of the third trader. Here we focus on the case where it is unambiguously known that $x_1 = +1$ and $x_2 = +1$.

Check the first row of the above table, where the third informed trader has a H signal. It can be verified that in this case, she is better off following her signal and trading $x_3 = +1$. The interesting case is in the second row of the table. The third informed trader has a L signal. If she follows her own signal, and chooses $x_3 = -1$, then the probability of her being correct is

$$\frac{\mathrm{P}(s_1 = H, s_2 = H, s_3 = L \mid \theta = L)\mathrm{P}(\theta = L)}{\mathrm{P}(s_1 = H, s_2 = H, s_3 = L)}$$
$$= 0.5q(1-q)^2/\mathrm{P}(s_1 = H, s_2 = H, s_3 = L).$$

On the other hand, if she ignores her signal and herds, choosing $x_3 = +1$, her probability of being correct is $\mathrm{P}(\theta = H \mid s_1 = H, s_2 = H, s_3 = L)$, which can be calculated as $0.5q^2(1-q)/\mathrm{P}(s_1 = H, s_2 = H, s_3 = L)$. Since $q > 0.5$, she can clearly improve her probability of being correct ex-post by herding and is better off following the two earlier traders in this case.

It may be noted that under our definition of herding, the third trader might also benefit from herding when she has a H signal, and the first two trades are unambiguously revealed to be $x_1 = -1$ and $x_2 = -1$. However, the externality d occurs only on the $\{+2, +2, +2\}$ price path, and the third trader cannot improve her expected profits from herding when $x_1 = -1$ and $x_2 = -1$. So, we do not focus on this case here. □

Proposition 2. *A herding equilibrium exists if, and only if,* $I > I^*$, *where*

$$I^* = \frac{3}{d(2q-1)}\left[\frac{q^2}{q^2 + (1-q)^2} - q\right] + \left[\frac{1}{q^2 + (1-q)^2} - 1\right], \quad \forall q \in (0.5, 1].$$

Proof. Under the established prices, we have to verify whether the conjectured behavior is indeed optimal for each of the informed traders. Specifically, we verify equilibrium behavior for each informed trader assuming equilibrium behavior on part of the other two.

Informed trader 1: IT1's expected profits are given by

$$E[\pi_1 \mid s_1, x_1 = +1] = E[v \mid s_1, x_1 = +1] - \tfrac{1}{2}[p_0^1 + p_2^1]$$

and

$$E[\pi_1 \mid s_1, x_1 = -1] = \tfrac{1}{2}[p_0^1 + p_{-2}^1] - E[v \mid s_1, x_1 = -1].$$

We need to develop expressions for $E[v \mid s_1, x_1 = +1]$ and $E[v \mid s_1, x_1 = -1]$. Using the probabilities and security payoffs conditioned on s_1, we have

$$E[v \mid s_1 = H, x_1 = +1] = q + \frac{d}{8}(2q-1)$$

and

$$E[v \mid s_1 = L, x_1 = +1] = (1-q).$$

Setting $d = 0$ in the above two expressions, we get the following:

$$E[v \mid s_1 = H, x_1 = -1] = q,$$
$$E[v \mid s_1 = L, x_1 = -1] = (1-q).$$

Trading strategy:

$$E[\pi_1 \mid s_1 = H, x_1 = +1] = q + \frac{d}{8}(2q-1) - \frac{1}{2}[p_0^1 + p_2^1]$$

and

$$E[\pi_1 \mid s_1 = H, x_1 = -1] = \tfrac{1}{2}[p_0^1 + p_{-2}^1] - q.$$

When $s_1 = H, x_1 = +1$ is preferred to $x_1 = -1$ if, and only if

$$q + \frac{d}{8}(2q-1) - \frac{1}{2}\left[\frac{1}{2} + q + \frac{d}{4}(2q-1)\right] > \frac{1}{2}\left[\frac{1}{2} + (1-q)\right] - q$$

$\Leftrightarrow q > \frac{1}{2}$, which is true under the assumptions of the model. So, $x_1 = +1$ is optimal when $s_1 = H$.

$$E[\pi_1 \mid s_1 = L, x_1 = +1] = (1-q) - \tfrac{1}{2}[p_0^1 + p_2^1]$$

and

$$E[\pi_1 \mid s_1 = L, x_1 = -1] = \tfrac{1}{2}[p_0^1 + p_{-2}^1] - (1-q).$$

When $s_1 = L, x_1 = -1$ is preferred to $x_1 = +1$ if, and only if

$$\frac{1}{2}\left[\frac{1}{2} + (1-q)\right] - (1-q) > (1-q) - \frac{1}{2}\left[\frac{1}{2} + q + \frac{d}{4}(2q-1)\right]$$
$$\Leftrightarrow (2q-1) > \frac{d}{8}(1-2q),$$

which is true $\forall q > \frac{1}{2}$. Thus, when $s_1 = L$, the first informed trader makes a trading loss if she trades $x_1 = +1$ rather than $x_1 = -1$. Also, she cannot gain on her existing inventory of securities by choosing $x_1 = +1$ over $x_1 = -1$, as $E[v \mid s_1 = L, x_1 = +1] = E[v \mid s_1 = L, x_1 = -1] = (1-q)$. Thus, $x_1 = -1$ is optimal when $s_1 = L$.

Informed trader 2: Here we have to consider all three cases that might have occurred in the first round of trading.

Case 1: $p^1 = p^1_{-2} (\Rightarrow s_1 = L)$.

Case 2: $p^1 = p^1_0 (\Rightarrow s_1 = H \text{ or } L)$.

Case 3: $p^1 = p^1_2 (\Rightarrow s_1 = H)$.

We show here details only for Case 3 (as there can be no temptation to go against one's signal in Cases 1 and 2).

IT2's expected trading profits are given by

$$\mathrm{E}[\pi_2 \,|\, p^1 = p^1_2, s_2, x_2 = +1] = \mathrm{E}[v \,|\, p^1 = p^1_2, s_2, x_2 = +1] - \tfrac{1}{2}[p^2_{2,0} + p^2_{2,2}]$$

and

$$\mathrm{E}[\pi_2 \,|\, p^1 = p^1_2, s_2, x_2 = -1] = \tfrac{1}{2}[p^2_{2,0} + p^2_{2,-2}] - \mathrm{E}[v \,|\, p^1 = p^1_2, s_2, x_2 = -1].$$

It can be shown that

$$\mathrm{E}[v \,|\, p^1 = p^1_2, s_2 = H, x_2 = +1] = \frac{q^2}{q^2 + (1-q)^2} + \frac{d}{4}\frac{2q-1}{q^2 + (1-q)^2}$$

and,

$$\mathrm{E}[v \,|\, p^1 = p^1_2, s_2 = L, x_2 = +1] = \tfrac{1}{2}.$$

Setting $d = 0$ in the above two expressions, we get the following:

$$\mathrm{E}[v \,|\, p^1 = p^1_2, s_2 = H, x_2 = -1] = \frac{q^2}{q^2 + (1-q)^2},$$

$$\mathrm{E}[v \,|\, p^1 = p^1_2, s_2 = L, x_2 = -1] = \tfrac{1}{2}.$$

Trading strategy: When $s_2 = H, x_2 = +1$ is preferred to $x_2 = -1$ if, and only if

$$\begin{aligned}
&\frac{q^2}{q^2 + (1-q)^2} + \frac{d}{4}\frac{2q-1}{q^2 + (1-q)^2} - \frac{1}{2}\left[q + \frac{q^2}{q^2 + (1-q)^2} + \frac{d}{2}\frac{2q-1}{q^2 + (1-q)^2}\right] \\
&\quad > \frac{1}{2}\left[q + \frac{1}{2}\right] - \frac{q^2}{q^2 + (1-q)^2} \\
&\quad \Leftrightarrow \frac{3}{2}\frac{q^2}{q^2 + (1-q)^2} > q + \frac{1}{4},
\end{aligned}$$

which is true $\forall q > \frac{1}{2}$. Thus, $x_2 = +1$ is optimal when $s_2 = H$. When $s_2 = L, x_2 = -1$ is preferred to $x_2 = +1$ if, and only if

$$\begin{aligned}
&\frac{1}{2}\left[q + \frac{1}{2}\right] - \frac{1}{2} > \frac{1}{2} - \frac{1}{2}\left[q + \frac{q^2}{q^2 + (1-q)^2} + \frac{d}{2}\frac{2q-1}{q^2 + (1-q)^2}\right] \\
&\qquad\qquad \Leftrightarrow \left[2q + \frac{q^2}{q^2 + (1-q)^2} - \frac{3}{2}\right] > \frac{d}{2}\frac{(1-2q)}{q^2 + (1-q)^2},
\end{aligned}$$

which is true $\forall q > \frac{1}{2}$ and $d > 0$. Thus, when $s_2 = L$, the IT2 makes a trading loss if she trades $x_2 = +1$ rather than $x_2 = -1$. Also, she cannot gain on her existing inventory of securities by choosing $x_2 = +1$ over $x_2 = -1$, as $\mathrm{E}[v \mid p^1 = p^1_2, s_2 = L, x_2 = +1] = \mathrm{E}[v \mid p^1 = p^1_2, s_2 = L, x_2 = -1]$. Thus, $x_2 = -1$ is optimal when $s_2 = L$.

Informed trader 3: Here we have to consider five cases, i.e., all distinct prices that could have been established in the second round of trading. All the possible cases are listed below:

a. $p^2 = p^2_a \equiv p^2_{-2,-2}$
b. $p^2 = p^2_b \equiv p^2_{-2,0} = p^2_{0,-2}$
c. $p^2 = p^2_c \equiv p^2_{-2,2} = p^2_{2,-2} = p^2_{0,0}$
d. $p^2 = p^2_d \equiv p^2_{2,0} = p^2_{0,2}$
e. $p^2 = p^2_e \equiv p^2_{2,2}$

In Cases 1–4 above, IT3's trade does not impact the value as the externality in value is likely to be created only in Case 5. It can be verified that in all these cases, IT3 trades according to her own signal i.e. she trades $x_3 = +1$ when $s_3 = H$ and $x_3 = -1$ when $s_3 = L$. We take up the more interesting Case 5 below.

Case 5: $p^2 = p^2_e = p^2_{2,2} (\Rightarrow s_1 = H, s_2 = H)$.

IT3's expected trading profits are given by

$$\mathrm{E}[\pi_3 \mid p^2 = p^2_e, s_3, x_3 = +1] = \mathrm{E}[v \mid p^2 = p^2_e, s_3, x_3 = +1] - \tfrac{1}{2}[p^3_{e,0} + p^3_{e,2}]$$

and

$$\mathrm{E}[\pi_3 \mid p^2 = p^2_e, s_3, x_3 = -1] = \tfrac{1}{2}[p^3_{e,0} + p^3_{e,-2}] - \mathrm{E}[v \mid p^2 = p^2_e, s_3, x_3 = -1],$$

$$\begin{aligned}\mathrm{E}[v \mid p^2 = p^2_e, s_3, x_3 = +1] = \tfrac{1}{2}\{&(1+d)\mathrm{P}(\theta = H \mid s_1 = H, s_2 = H, s_3) \\ &- d\mathrm{P}(\theta = L \mid s_1 = H, s_2 = H, s_3) \\ &+ \mathrm{P}(\theta = H \mid s_1 = H, s_2 = H, s_3)\}.\end{aligned}$$

So,

$$\mathrm{E}[v \mid p^2 = p^2_e, s_3 = H, x_3 = +1] = \frac{q^3}{q^3 + (1-q)^3} + \frac{d}{2}\frac{[q^3 - (1-q)^3]}{q^3 + (1-q)^3}$$

and

$$\mathrm{E}[v \mid p^2 = p^2_e, s_3 = L, x_3 = +1] = q + \frac{d}{2}(2q - 1).$$

When IT3 invests $x_3 = -1$, she knows that the externality d will not be created.

So,

$$\mathrm{E}[v \mid p^2 = p^2_e, s_3 = H, x_3 = -1] = \frac{q^3}{q^3 + (1-q)^3}$$

$$\mathrm{E}[v \mid p^2 = p^2_e, s_3 = L, x_3 = -1] = q.$$

Trading strategy: When $s_3 = H, x_3 = +1$ is preferred to $x_3 = -1$ if, and only if

$$\frac{2q^3}{q^3 + (1-q)^3} + \frac{d}{2}\frac{[q^3 - (1-q)^3]}{q^3 + (1-q)^3} > \frac{3}{2}\frac{q^2}{q^2 + (1-q)^2} + \frac{d}{2}\frac{(2q-1)}{q^2 + (1-q)^2} + \frac{q}{2}$$

which is true $\forall q > \frac{1}{2}$ and $d > 0$. Thus, $x_3 = +1$ is optimal when $s_3 = H$. When $s_3 = L$, $x_3 = +1$ is preferred to $x_3 = -1$ (i.e., IT3 herds) if, and only if

$$\frac{3q}{2} - \frac{3}{2}\frac{q^2}{q^2 + (1-q)^2} > \frac{d}{2}(2q-1)\left[\frac{1}{q^2 + (1-q)^2} - 1\right]$$

which fails for $q > \frac{1}{2}$ and $d > 0$. Thus, when $s_3 = L$, IT3 makes a trading loss if she trades $x_3 = +1$ rather than $x_3 = -1$ (i.e., if she herds). The quantum of this trading loss is

$$\frac{3}{2}\left[\frac{q^2}{q^2 + (1-q)^2} - q\right] + \frac{d}{2}(2q-1)\left[\frac{1}{q^2 + (1-q)^2} - 1\right].$$

It is clear that in the absence of trader inventories, herding is not an equilibrium strategy. However, when informed traders have inventories in the security, IT3 can gain on her inventory by herding since

$$\begin{aligned} \mathrm{E}[v \mid p^2 = p_e^2, s_3 = L, x_3 = +1] = q + \frac{d}{2}(2q-1) &> \mathrm{E}[v \mid p^2 = p_e^2, s_3 = L, x_3 = -1] \\ &= q, \quad \forall d > 0. \end{aligned}$$

By herding, IT3 can gain an extra $d(2q-1)/2$ per each security in her inventory.

Thus, a necessary and sufficient condition for herding to be supported in equilibrium is

$$I\frac{d}{2}(2q-1) > \frac{3}{2}\left[\frac{q^2}{q^2 + (1-q)^2} - q\right] + \frac{d}{2}(2q-1)\left[\frac{1}{q^2 + (1-q)^2} - 1\right], \tag{A.1}$$

which is true (since $q > \frac{1}{2}$ is assumed)

$$\Leftrightarrow I > I^* \equiv \frac{3}{d(2q-1)}\left[\frac{q^2}{q^2 + (1-q)^2} - q\right] + \left[\frac{1}{q^2 + (1-q)^2} - 1\right]. \qquad \square$$

Corollary 1. a. *With totally uninformative signals, i.e., with $q = 0.5$, a herding equilibrium is not possible.*

b. *For given d, I^* decreases in q, $\forall q > 0.5$.*

c. $\forall q \in (0.5, 1], I^*$ *decreases in d, $\forall d > 0$.*

Proof. (a) Substitute $q = 0.5$ into Condition (A.1) above. Clearly the inequality is not strictly satisfied. Stated another way, this means that it is not possible for traders with totally uninformative signals to start and maintain herding in equilibrium.

(b) This part is illustrated in Fig. A1. The figure shows that herding is more likely when the quality of the informed traders' information is higher. For higher quality information, later informed traders are more likely to trust the information of those who have already traded and are more comfortable disregarding their own signal. For the same reason, the critical level of inventory needed to support herding is negatively related to information quality. With higher information quality, the information loss from herding is less. Thus prices remain close to expected prices

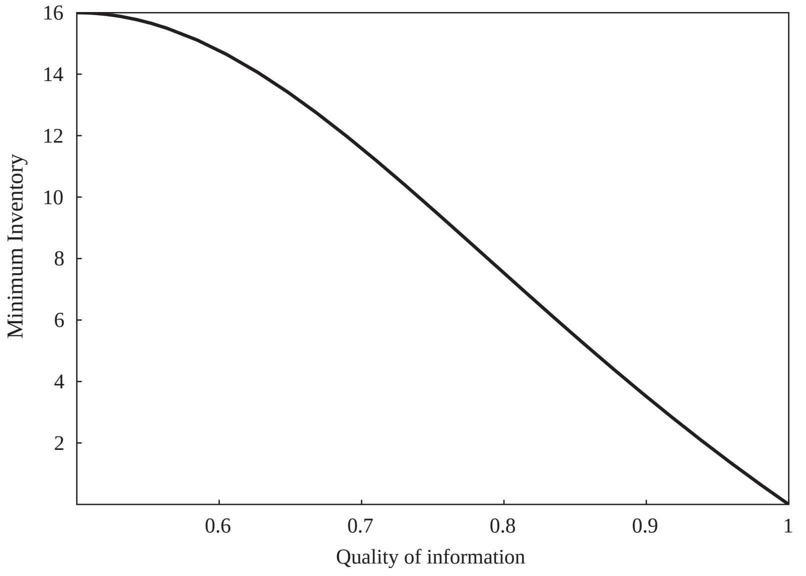


Fig. A1. Minimum inventory bound for herding equilibrium ($d = 0.1$). I^* is the minimum bound on inventory for a herding equilibrium to exist. It can be seen that holding the value of d constant, as q increases and signals become more informative, smaller levels of inventories are needed to start herding.

conditional on no information loss. Since trading losses from herding are less, smaller inventory is needed to recover the losses.

(c) See Fig. A2 for an illustration of this part of the corollary. The intuition here is that holding q constant, as d increases, the potential gains to herding increase, and hence smaller levels of inventory are sufficient to start herding. Not surprisingly, herding is more likely when the feedback effect is larger. □

A.2. Prices under no-herding: Expressions in Table 3

Price setting: *round* 1 *of trading*: Prices in this equilibrium are derived in a manner identical to those in the herding equilibrium.

$$p^{n1}_{-2} = \mathrm{P}(\theta = H \mid s_1 = L) = (1 - q),$$
$$p^{n1}_{0} = \tfrac{1}{2}[\mathrm{P}(\theta = H \mid s_1 = H) + \mathrm{P}(\theta = H \mid s_1 = L)] = \tfrac{1}{2},$$
$$p^{n1}_{2} = q + \frac{d}{4}[q^3 - (1 - q)^3].$$

Given the above three prices, the price at time 0 in this equilibrium is

$$\begin{aligned} p^{n0} &= p^{n1}_{-2}\mathrm{P}(y_1 = -2) + p^{n1}_{0}\mathrm{P}(y_1 = 0) + p^{n1}_{2}\mathrm{P}(y_1 = 2),\\ &= \frac{1}{2} + \frac{d}{16}[q^3 - (1 - q)^3]. \end{aligned}$$

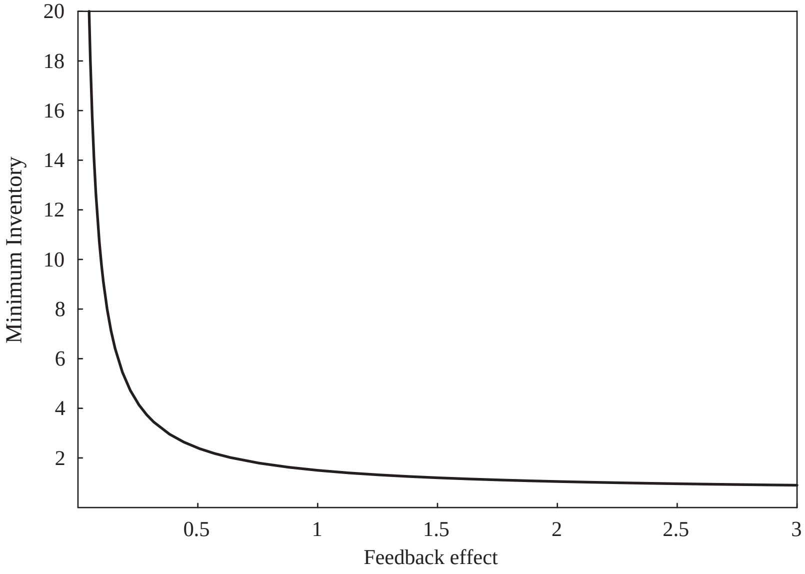


Fig. A2. Minimum inventory bound for herding equilibrium ($q = 0.75$). I^* is the minimum bound on inventory for a herding equilibrium to exist. It can be seen that holding the value of q constant, as d increases and the potential gains to herding increase, smaller levels of inventories are needed to start herding.

Price setting: *round* 2 *of trading*: Prices p_a^{n2} through p_d^{n2} can be derived exactly like p_a^2 through p_d^2 of the herding case. The interesting case is $p_e^{n2} = p_{2,2}^{n2}$, which is derived below.

$$\begin{aligned} p_e^{n2} = p_{2,2}^{n2} &= \mathrm{P}(s_3 = L \ \& \ \theta = H \mid s_1 = H \ \& \ s_2 = H) \\ &+ \mathrm{P}(s_3 = H \ \& \ y_3 = 0 \ \& \ \theta = H \mid s_1 = H \ \& \ s_2 = H) \\ &+ (1+d)\mathrm{P}(s_3 = H \ \& \ y_3 = 2 \ \& \ \theta = H \mid s_1 = H \ \& \ s_2 = H) \\ &- d\mathrm{P}(s_3 = H \ \& \ y_3 = 2 \ \& \ \theta = L \mid s_1 = H \ \& \ s_2 = H) \\ &= \frac{q^2}{q^2 + (1-q)^2} + \frac{d}{2} \frac{[q^3 - (1-q)^3]}{[q^2 + (1-q)^2]}. \end{aligned}$$

Price setting: *round* 3 *of trading*: Prices $p_{a,-2}^{n3}$ through $p_{e,-2}^{n3}$ can be derived exactly like $p_{a,-2}^3$ through $p_{e,-2}^3$ in the herding case. The interesting cases are $p_{e,0}^{n3}$ and $p_{e,2}^{n3}$, which are derived below.

$$p_{e,0}^{n3} = p_{2,2,0}^{n3} = \mathrm{P}(\theta = H \mid s_1 = H \ \& \ s_2 = H \ \& \ s_3 = H \ or \ L) = \frac{q^2}{q^2 + (1-q)^2},$$

$$\begin{aligned} p^{n3}_{e,2} = p^{n3}_{2,2,2} &= (1+d)\mathrm{P}(\theta = H \mid s_1 = H \ \&\ s_2 = H \ \&\ s_3 = H), \\ &\quad - d\mathrm{P}(\theta = L \mid s_1 = H \ \&\ s_2 = H \ \&\ s_3 = H) \\ &= \frac{q^3}{q^3 + (1-q)^3} + d\frac{[q^3 - (1-q)^3]}{[q^3 + (1-q)^3]}. \end{aligned}$$

Proposition 3. *A no-herding equilibrium exists if, and only if* $I < \tilde{I}$.

Proof. *Informed trader* 1: IT1's expected profits are given by

$$\mathrm{E}[\pi_1 \mid s_1, x_1 = +1] = \mathrm{E}[v \mid s_1, x_1 = +1] - \tfrac{1}{2}[p^{n1}_0 + p^{n1}_2]$$

and

$$\mathrm{E}[\pi_1 \mid s_1, x_1 = -1] = \tfrac{1}{2}[p^{n1}_0 + p^{n1}_{-2}] - \mathrm{E}[v \mid s_1, x_1 = -1].$$

We need expressions for $\mathrm{E}[v \mid s_1, x_1 = +1]$ and $\mathrm{E}[v \mid s_1, x_1 = -1]$. Using the probabilities and security payoffs conditioned on s_1, we have

$$\mathrm{E}[v \mid s_1 = H, x_1 = +1] = q + \frac{d}{8}[q^3 - (1-q)^3]$$

and

$$\mathrm{E}[v \mid s_1 = L, x_1 = +1] = (1-q) + \frac{d}{8}[3q^2 - 2q^3 - q].$$

Setting $d = 0$ in the above two expressions, we get the following:

$$\begin{aligned} &\mathrm{E}[v \mid s_1 = H, x_1 = -1] = q, \\ &\mathrm{E}[v \mid s_1 = L, x_1 = -1] = (1-q). \end{aligned}$$

Trading strategy:

$$\mathrm{E}[\pi_1 \mid s_1 = H, x_1 = +1] = q + \frac{d}{8}[q^3 - (1-q)^3] - \frac{1}{2}[p^{n1}_0 + p^{n1}_2]$$

and

$$\mathrm{E}[\pi_1 \mid s_1 = H, x_1 = -1] = \tfrac{1}{2}[p^{n1}_0 + p^{n1}_{-2}] - q.$$

When $s_1 = H, x_1 = +1$ is preferred to $x_1 = -1$ if, and only if

$$\begin{aligned} &q + \frac{d}{8}[q^3 - (1-q)^3] - \frac{1}{2}\left[\frac{1}{2} + q + \frac{d}{4}[q^3 - (1-q)^3]\right] > \frac{1}{2}\left[\frac{1}{2} + (1-q)\right] - q \\ &\quad \Leftrightarrow q > \frac{1}{2} \end{aligned}$$

which is true under the assumptions of the model. Thus, $x_1 = +1$ is optimal when $s_1 = H$.

$$\mathrm{E}[\pi_1 \mid s_1 = L, x_1 = +1] = (1-q) + \frac{d}{8}[3q^2 - 2q^3 - q] - \frac{1}{2}[p^{n1}_0 + p^{n1}_2]$$

and

$$\mathrm{E}[\pi_1 \,|\, s_1 = L, x_1 = -1] = \tfrac{1}{2}[p_0^{n1} + p_{-2}^{n1}] - (1-q).$$

When $s_1 = L, x_1 = -1$ is preferred to $x_1 = +1$ if, and only if

$$\frac{1}{2}\left[\frac{1}{2} + (1-q)\right] - (1-q) > (1-q) + \frac{d}{8}[3q^2 - 2q^3 - q]$$
$$-\frac{1}{2}\left[\frac{1}{2} + q + \frac{d}{4}(q^3 - (1-q)^3)\right]$$
$$\Leftrightarrow (2q-1) > \frac{d}{8}(6q^2 - 4q^3 - 4q + 1)$$

which is true $\forall q > \frac{1}{2}$ and $d > 0$. Thus, when $s_1 = L$, trading $x_1 = -1$ instead of trading $x_1 = +1$ yields IT1 an expected profit of

$$(2q-1) - \frac{d}{8}(6q^2 - 4q^3 - 4q + 1).$$

However, trading $x_1 = +1$ rather than $x_1 = -1$ increments the expected value of each unit of inventory by $d/8[3q^2 - 2q^3 - q]$. (This is the difference between $\mathrm{E}[v \,|\, s_1 = L, x_1 = +1]$ and $\mathrm{E}[v \,|\, s_1 = L, x_1 = -1]$). Thus, we require the inventory of the first informed trader to be such that her potential expected value addition to inventory from deviating from the equilibrium behavior (and trading $x_1 = +1$) is outweighed by her trading profits from following the equilibrium strategy, i.e., we need that:

$$I\frac{d}{8}[3q^2 - 2q^3 - q] < (2q-1) - \frac{d}{8}(6q^2 - 4q^3 - 4q + 1)$$
$$\Leftrightarrow I < I_1 \equiv \left[\frac{8(2q-1)}{d(3q^2 - 2q^3 - q)} - \frac{(6q^2 - 4q^3 - 4q + 1)}{(3q^2 - 2q^3 - q)}\right].$$

Informed trader 2: Here we have to consider all three cases that might have occurred in the first round of trading. Cases 1 and 2 where $p^{n1} = p_{-2}^{n1}$ and $p^{n1} = p_0^{n1}$ are not considered here. We deal only with the more interesting Case 3.

Case 3: $p^{n1} = p_2^{n1} (\Rightarrow s_1 = H)$.

IT2's expected trading profits under both possible trades are given by

$$\mathrm{E}[\pi_2 \,|\, p^{n1} = p_2^{n1}, s_2, x_2 = +1] = \mathrm{E}[v \,|\, p^{n1} = p_2^{n1}, s_2, x_2 = +1] - \tfrac{1}{2}[p_{2,0}^{n2} + p_{2,2}^{n2}]$$

$$\mathrm{E}[\pi_2 \,|\, p^{n1} = p_2^{n1}, s_2, x_2 = -1] = \tfrac{1}{2}[p_{2,0}^{n2} + p_{2,-2}^{n2}] - \mathrm{E}[v \,|\, p^{n1} = p_2^{n1}, s_2, x_2 = -1].$$

It can be shown that

$$\mathrm{E}[v \,|\, p^{n1} = p_2^{n1}, s_2 = H, x_2 = +1] = \frac{q^2}{q^2 + (1-q)^2} + \frac{d}{4}\frac{[q^3 - (1-q)^3]}{q^2 + (1-q)^2}$$

and,

$$\mathrm{E}[v \,|\, p^{n1} = p_2^{n1}, s_2 = L, x_2 = +1] = \frac{1}{2} + \frac{d}{8}(2q-1).$$

Setting $d = 0$ in the above two expressions, we get the following:

$$\mathrm{E}[v \mid p^{n1} = p_2^{n1}, s_2 = H, x_2 = -1] = \frac{q^2}{q^2 + (1-q)^2},$$
$$\mathrm{E}[v \mid p^{n1} = p_2^{n1}, s_2 = L, x_2 = -1] = \tfrac{1}{2}.$$

Trading strategy: When $s_2 = H, x_2 = +1$ is preferred to $x_2 = -1$ if, and only if

$$\begin{aligned}
&\frac{q^2}{q^2+(1-q)^2} + \frac{d}{4}\frac{[q^3-(1-q)^3]}{q^2+(1-q)^2} - \frac{1}{2}\left[q + \frac{q^2}{q^2+(1-q)^2} + \frac{d}{2}\frac{[q^3-(1-q)^3]}{q^2+(1-q)^2}\right] \\
&\quad > \frac{1}{2}\left[q + \frac{1}{2}\right] - \frac{q^2}{q^2+(1-q)^2} \\
&\quad \Leftrightarrow \frac{3}{2}\left[\frac{q^2}{q^2+(1-q)^2}\right] > q + \frac{1}{4},
\end{aligned}$$

which is true $\forall q > \frac{1}{2}$. Thus, $x_2 = +1$ is optimal when $s_2 = H$.

When $s_2 = L, x_2 = -1$ is preferred to $x_2 = +1$ if, and only if

$$\begin{aligned}
&\frac{1}{2}\left[q + \frac{1}{2}\right] - \frac{1}{2} > \frac{1}{2} + \frac{d}{8}(2q-1) - \frac{1}{2}\left[q + \frac{q^2}{q^2+(1-q)^2} + \frac{d}{2}\frac{[q^3-(1-q)^3]}{q^2+(1-q)^2}\right] \\
&\quad \Leftrightarrow \left[q + \frac{q^2}{2[q^2+(1-q)^2]} - \frac{3}{4}\right] < \frac{d}{8}\left[(2q-1) - \frac{2[q^3-(1-q)^3])}{q^2+(1-q)^2}\right],
\end{aligned}$$

which is true $\forall q > \frac{1}{2}$ and $d > 0$.

However, trading $x_2 = +1$ rather than $x_2 = -1$ increments the expected value of each unit of inventory by $d/8(2q-1)$. (This is the difference between $\mathrm{E}[v \mid p^{n1} = p_2^{n1}, s_2 = L, x_2 = +1]$ and $\mathrm{E}[v \mid p^{n1} = p_2^{n1}, s_2 = L, x_2 = -1]$.) Thus, we require the inventory of the second informed trader to be such that her potential expected value addition to inventory from deviating from the equilibrium behavior (and trading $x_2 = +1$) is outweighed by her trading profits from following the equilibrium strategy, i.e., we need that

$$\begin{aligned}
&I\frac{d}{8}(2q-1) \\
&\quad < \left(q - \frac{3}{4}\right) + \frac{q^2}{2[q^2+(1-q)^2]} - \frac{d}{8}\left[(2q-1) - \frac{2[q^3-(1-q)^3])}{q^2+(1-q)^2}\right] \\
&\quad \Leftrightarrow I < I_2 \\
&\quad = \left[\frac{8q-6}{d(2q-1)} + \frac{4q^2}{d(2q-1)[q^2+(1-q)^2]} + \frac{2[q^3-(1-q)^3])}{(2q-1)[q^2+(1-q)^2]} - 1\right].
\end{aligned}$$

Informed trader 3: Here, as with herding, we have to consider five cases, i.e., all distinct prices that could have been established in the second round of trading. Cases 1–4 above are identical to those in the herding equilibrium. It can be verified that in all these cases, IT3 trades according to her own signal, i.e., she trades $x_3 = +1$ when

$s_3 = H$ and $x_3 = -1$ when $s_3 = L$. We take up the more interesting Case 5 below:

Case 5: $p^{n2} = p_e^{n2} = p_{2,2}^{n2} (\Rightarrow s_1 = H, s_2 = H)$.

IT3's expected trading profits are given by

$$\mathrm{E}[\pi_3 \,|\, p^{n2} = p_e^{n2}, s_3, x_3 = +1] = \mathrm{E}[v \,|\, p^{n2} = p_e^{n2}, s_3, x_3 = +1] - \tfrac{1}{2}[p_{e,0}^{n3} + p_{e,2}^{n3}]$$

and

$$\mathrm{E}[\pi_3 \,|\, p^{n2} = p_e^{n2}, s_3, x_3 = -1] = \tfrac{1}{2}[p_{e,0}^{n3} + p_{e,-2}^{n3}] - \mathrm{E}[v \,|\, p^{n2} = p_e^{n2}, s_3, x_3 = -1].$$

It can be shown that

$$\mathrm{E}[v \,|\, p^{n2} = p_e^{n2}, s_3 = H, x_3 = +1] = \frac{q^3}{q^3 + (1-q)^3} + \frac{d}{2}\frac{[q^3 - (1-q)^3]}{q^3 + (1-q)^3},$$

$$\mathrm{E}[v \,|\, p^{n2} = p_e^{n2}, s_3 = L, x_3 = +1] = q + \frac{d}{2}(2q - 1).$$

Setting $d = 0$ in the above expressions, we get

$$\mathrm{E}[v \,|\, p^{n2} = p_e^{n2}, s_3 = H, x_3 = -1] = \frac{q^3}{q^3 + (1-q)^3},$$

$$\mathrm{E}[v \,|\, p^{n2} = p_e^{n2}, s_3 = L, x_3 = -1] = q.$$

Trading strategy: When $s_3 = H, x_3 = +1$ is preferred to $x_3 = -1$ if, and only if

$$\begin{aligned}
&\frac{q^3}{q^3 + (1-q)^3} + \frac{d}{2}\frac{[q^3 - (1-q)^3]}{q^3 + (1-q)^3} \\
&\quad - \frac{1}{2}\left[\frac{q^2}{[q^2 + (1-q)^2]} + \frac{q^3}{q^3 + (1-q)^3} + \frac{d[q^3 - (1-q)^3]}{q^3 + (1-q)^3}\right] \\
&\quad > \frac{1}{2}\left[q + \frac{q^2}{q^2 + (1-q)^2}\right] - \frac{q^3}{q^3 + (1-q)^3} \\
&\quad \Leftrightarrow \frac{3}{2}\left[\frac{q^3}{q^3 + (1-q)^3}\right] > \left[\frac{q^2}{q^2 + (1-q)^2}\right] + \frac{q}{2},
\end{aligned}$$

which is true $\forall q > \frac{1}{2}$ and $d > 0$. Thus, $x_3 = +1$ is optimal when $s_3 = H$.

When $s_3 = L$, $x_3 = -1$ is preferred to $x_3 = +1$ if, and only if

$$\begin{aligned}
&\frac{1}{2}\left[q + \frac{q^2}{q^2 + (1-q)^2}\right] - q \\
&\quad > q + \frac{d}{2}(2q - 1) \\
&\qquad - \frac{1}{2}\left[\frac{q^2}{q^2 + (1-q)^2} + \frac{q^3}{q^3 + (1-q)^3} + \frac{d[q^3 - (1-q)^3]}{q^3 + (1-q)^3}\right] \\
&\quad \Leftrightarrow \left[\frac{q^2}{q^2 + (1-q)^2} + \frac{q^3}{2[q^3 + (1-q)^3]} - \frac{3q}{2}\right] > \frac{d}{2}(2q - 1) - \frac{d}{2}\frac{[q^3 - (1-q)^3]}{q^3 + (1-q)^3}
\end{aligned}$$

which is true for $q > \frac{1}{2}$ and $d > 0$. Therefore, trading $x_3 = -1$ instead of $x_3 = +1$ yields IT3 a profit of

$$\frac{q^2}{q^2 + (1-q)^2} + \frac{q^3}{2[q^3 + (1-q)^3]} - \frac{3q}{2} - \frac{d}{2}(2q-1) + \frac{d}{2}\frac{[q^3 - (1-q)^3]}{q^3 + (1-q)^3}.$$

However, trading $x_3 = +1$ rather than $x_3 = -1$ increments the expected value of each unit of inventory by $d/2(2q-1)$. Thus, we require the inventory of IT3 to be such that her potential expected value addition to inventory from deviating from the equilibrium behavior (and trading $x_3 = +1$) is outweighed by her trading profit from following the equilibrium strategy, i.e., we need that

$$\begin{aligned} I\frac{d}{2}(2q-1) < {} & \frac{q^2}{q^2 + (1-q)^2} + \frac{q^3}{2[q^3 + (1-q)^3]} - \frac{3q}{2} - \frac{d}{2}(2q-1) \\ & + \frac{d}{2}\frac{[q^3 - (1-q)^3]}{q^3 + (1-q)^3} \\ \Leftrightarrow I < I_3 \equiv {} & \left[\frac{2q^2}{d(2q-1)[q^2 + (1-q)^2]} + \frac{q^3}{d(2q-1)[q^3 + (1-q)^3]} \right. \\ & \left. - \frac{3q}{d(2q-1)} + \frac{[q^3 - (1-q)^3]}{(2q-1)[q^3 + (1-q)^3]} - 1\right]. \end{aligned}$$

Since we have assumed symmetric informed traders (with identical inventories), a sufficient condition for a no-herding equilibrium to exist is that $I < \tilde{I} = \min(I_1, I_2, I_3)$. Since the order of trading is randomly determined, this condition is also necessary. It

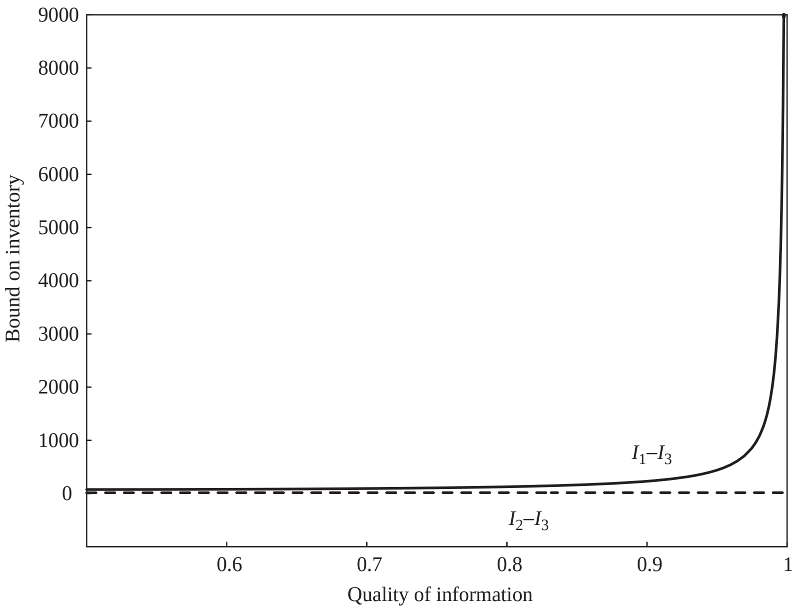


Fig. A3. Comparison of inventory bounds for no-herding ($d = 0.4$).

can be verified that I_3 is the smallest of I_1, I_2 and I_3. Hence, $\tilde{I} = I_3$. This intuitively makes sense, as the third informed trader, knowing about the two H signals in the two earlier rounds of trading, has the strongest temptation to herd, even though her own signal is an L signal. Thus, the constraint on her inventory is much more binding than those on the earlier two traders. It can be seen from Fig. A3 that $I_1 - I_3$ and $I_2 - I_3$ are both positive over the support of q that we consider in this model. Another interesting observation from Fig. A3 is regarding the shape of $I_1 - I_3$, which is sloping upward as q increases. The reason is that the first informed trader is increasingly reluctant to start herding by trading against her own signal as it becomes more informative. □

References

Allen, F., Gorton, G., 1993. Churning bubbles. Review of Economic Studies 60, 813–836.

Avery, C., Zemsky, P., 1998. Multidimensional uncertainty and herd behavior in financial markets. American Economic Review 88, 724–748.

Barberis, N., Shleifer, A., Vishny, R., 1998. A model of investor sentiment. Journal of Financial Economics 49, 307–343.

Bernanke, B., 1983. Nonmonetary effects of the financial crisis in propagation of the great depression. American Economic Review 73, 257–276.

Bernanke, B., Gertler, M., 1989. Agency costs, net worth, and business fluctuations. American Economic Review 79, 14–31.

Bikhchandani, S., Hirshleifer, D., Welch, I., 1992. A theory of fads, fashions, customs and cultural change as informational cascades. Journal of Political Economy 100, 992–1026.

Cho, I.-K., Kreps, D.M., 1987. Signaling games and stable equilibria. Quarterly Journal of Economics 102, 179–221.

Daniel, K., Hirshleifer, D., Subrahmanyam, A., 1998. Investor psychology and security market under- and overreaction. Journal of Finance 61, 1839–1886.

DeLong, J.B., Shleifer, A., Summers, L.H., Waldmann, R.J., 1990. Positive feedback investment strategies and destabilizing rational speculation. Journal of Finance 45, 379–395.

Eckstein, O., Sinai, A., 1986. The mechanisms of the business cycle in the post-war era. In: Gordon R.J. (Ed.), The American Business Cycle: Continuity and Change, Vol. 25. University of Chicago Press, Chicago, pp. 39–105.

Falkenstein, E.G., 1996. Preferences for stock characteristics as revealed by mutual fund portfolio holdings. Journal of Finance 51, 111–135.

Fisher, I., 1933. The debt-deflation theory of the great depression. Econometrica 1, 337–357.

Friend, I., Blume, M., Crockett, J., 1970. Mutual Funds and other Institutional Investors. McGraw-Hill, New York, NY.

Glosten, L., Milgrom, P., 1985. Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. Journal of Financial Economics 14, 71–100.

Grinblatt, M., Titman, S., Wermers, R., 1995. Momentum investment strategies, portfolio performance and herding: a study of mutual fund behavior. American Economic Review 85, 1088–1105.

Grossman, S., Stiglitz, J., 1980. On the impossibility of informationally efficient markets. American Economic Review 71, 393–408.

Hong, H., Stein, J.C., 1999. A unified theory of underreaction, momentum trading and overreaction in asset markets. Journal of Finance 54, 2143–2184.

Hubbard, R.G., 1998. Capital market imperfections and investment. Journal of Economic Literature 36, 193–225.

Khanna, N., 1998. Optimal contracting with moral hazard and cascading. Review of Financial Studies 11, 559–596.

Khanna, N., Slezak, S.L., Bradley, M., 1994. Insider trading, outside search, and resource allocation: why firms and society may disagree on insider trading restrictions. Review of Financial Studies 7, 575–608.
Kiyotaki, N., Moore, J., 1997. Credit cycles. Journal of Political Economy 105, 211–248.
Kumar, P., Seppi, D., 1992. Futures manipulation with Cash Settlement. Journal of Finance 47, 1485–1502.
Kyle, A.S., 1985. Continuous auctions and insider trading. Econometrica 53, 1315–1336.
Lakonishok, J., Shleifer, A., Vishny, R., 1992. The impact of institutional trading on stock prices. Journal of Financial Economics 32, 23–43.
Leland, H., 1992. Insider trading: should it be prohibited? Journal of Political Economy 100, 859–887.
LeRoy, S., Porter, R., 1981. The present value relation: tests based on variance bounds. Econometrica 49, 555–577.
Nofsinger, J.R., Sias, R.W., 1999. Herding and feedback trading by institutional and individual investors. Journal of Finance 54, 2263–2295.
Shiller, R., 1981. Do stock prices move too much to be justified by subsequent changes in dividends? American Economic Review 71, 421–436.
Subrahmanyam, A., 1991. Risk aversion, market liquidity and price efficiency. Review of Financial Studies 4, 417–441.
Subrahmanyam, A., Titman, S., 2001. Feedback from stock prices to cash flows. Journal of Finance 56, 2389–2413.
Wermers, R., 1999. Mutual fund herding and the impact on stock prices. Journal of Finance 54, 581–622.